



# Recent Progress in the High-Gain FEL Theory

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## Introduction

- An exciting path to future x-ray sources: a high-gain FEL operated in SASE mode
- FEL theory well developed in the exponential growth regime (energy spread, emittance, diffraction, guiding)
- Tremendous progress in high-gain experiments  
wavelength down to  $< 100$  nm  
saturation achieved
- Stimulate new developments in high-gain FEL theory, some aspects are discussed in this talk  
(mostly based on collaborative work with K.-J. Kim)

# Overview

## Start-up stage

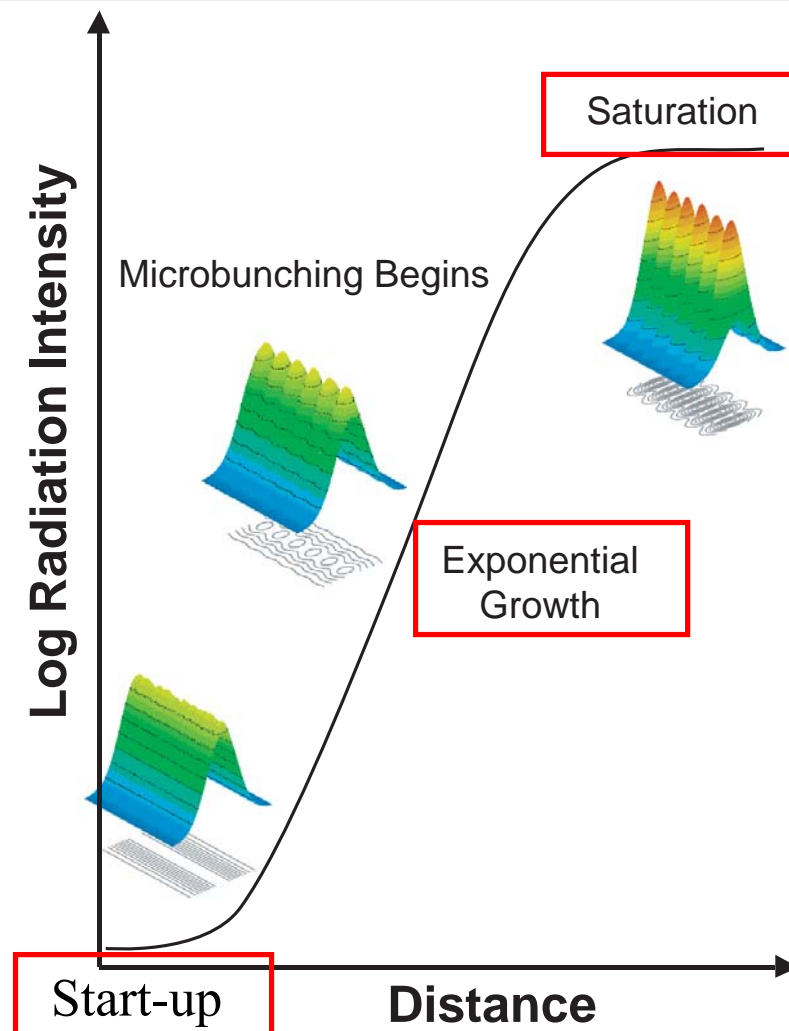
External signal or spontaneous radiation starts to interact with the e-beam resonantly at undulator  $\lambda$

Energy modulation  $\rightarrow$  density modulation (microbunching)  $\rightarrow$  coherent radiation at  $\lambda$

**Exponential growth ( $L_G$ )**

At sufficiently high power, electrons fully microbunched with large energy spread  $\rightarrow$  reach

**Saturation ( $P_{\text{sat}}$ )**





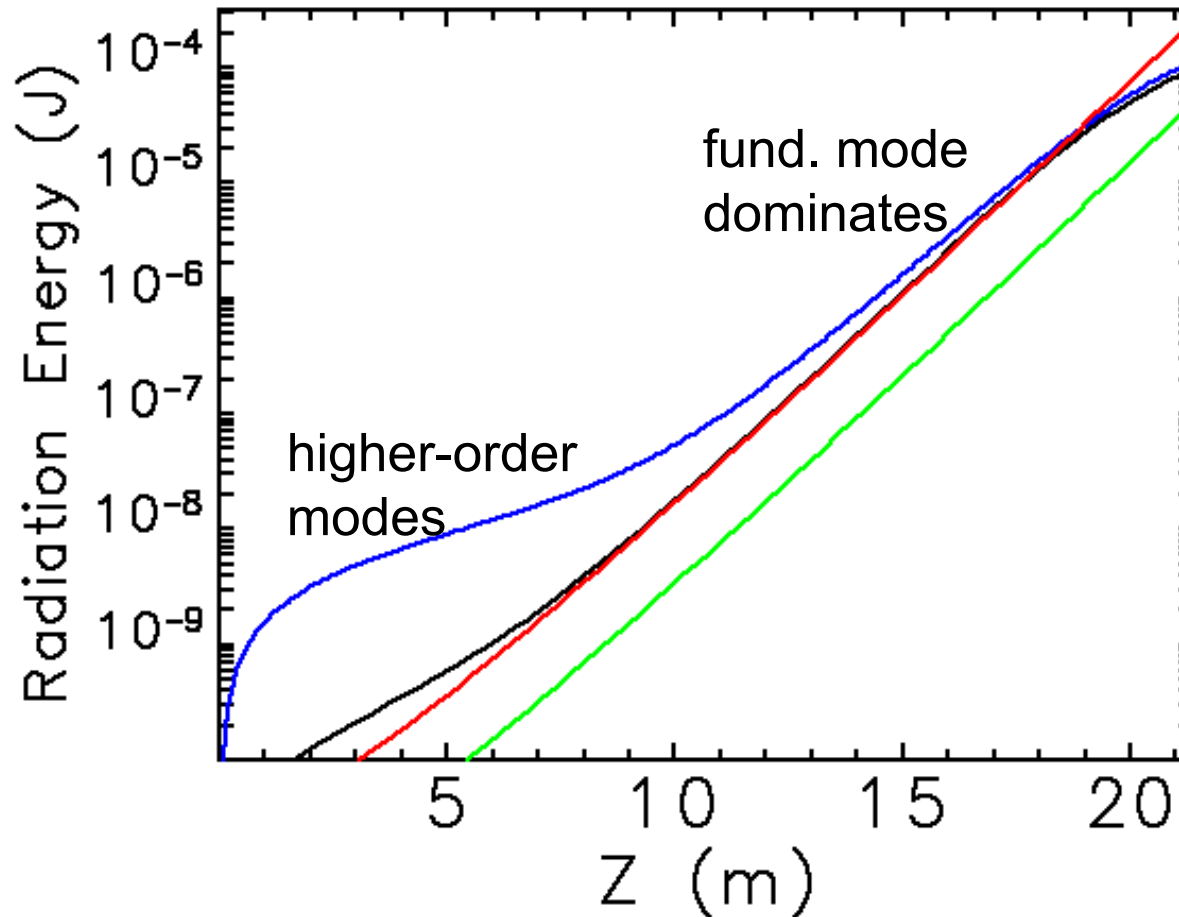
## Start-up Process

- Spontaneous emission excites many transverse modes
- FEL instability favors a particular (fundamental) mode
- ➔ proper modal decomposition for initial value problem

$$\left( \frac{dP}{d\omega} \right)_{\text{fund. mode}} = g_A \left[ \left( \frac{dP}{d\omega} \right)_{\text{noise}} + \left( \frac{dP}{d\omega} \right)_{\text{signal}} \right] \exp \left( \frac{Z}{L_G} \right)$$

- Effective start-up noise (for SASE): coherent fraction of the spontaneous emission over the first two  $L_G$  and can increase with energy spread and emittance through  $L_G$
- 2D Solution determines the radiation energy level in exponential gain regime

# Comparison with Time-dependent Codes



GENESIS 3D

GINGER 2D

Theory 2D

Theory 1D\*

\* 2D  $L_G$  used



## Basic Transverse and Temporal Properties

- Diffraction + Gain → transverse mode selection  
⇒ fundamental mode dominates (gain guiding)  
⇒ high transverse coherence

- SASE is a chaotic light temporally

$$\text{Coherence length} = \frac{c}{2\sigma_\omega} \approx \frac{\lambda}{4\pi} \sqrt{\frac{N_u}{\rho}} \ll \text{Bunch length}$$

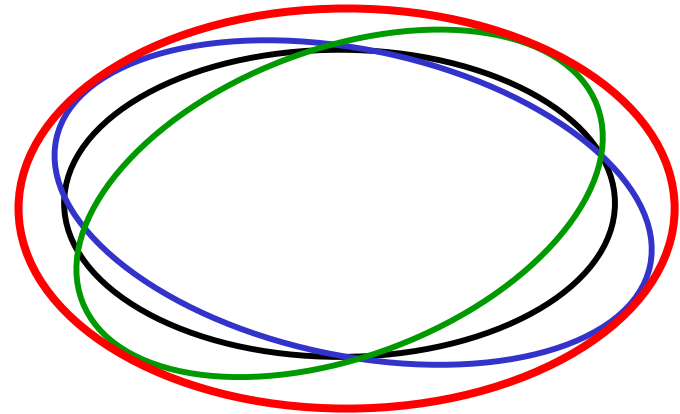
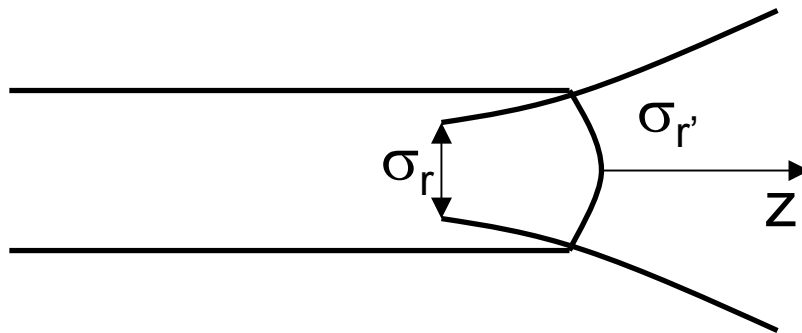
Intensity fluctuation

$$\frac{\Delta I}{I} = \frac{1}{\sqrt{M}}, \text{ where } M = \frac{\text{bunch length}}{\text{coherence length}}$$

- Statistical fluctuation is large for long-wavelength exps, but much smaller for XFELs (dominated by jitter etc...)

## Transverse and Temporal Properties: Interplay

- Transverse coherence somewhat affected by “large” SASE bandwidth (Saldin et al.)
- Different fundamental modes for different frequencies
- FEL fundamental mode and its transverse phase space

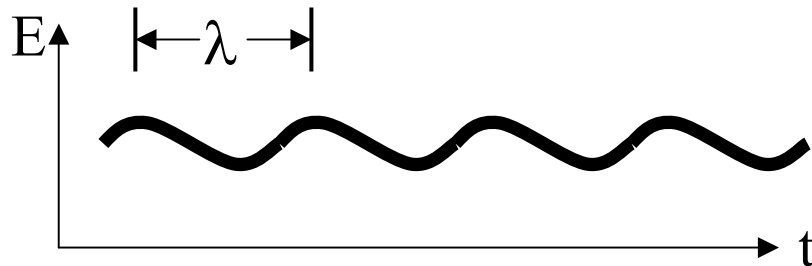


- Smearing of radiation phase space ellipses reduces transverse coherence: LEUTL ~ 90%, LCLS ~ 97%

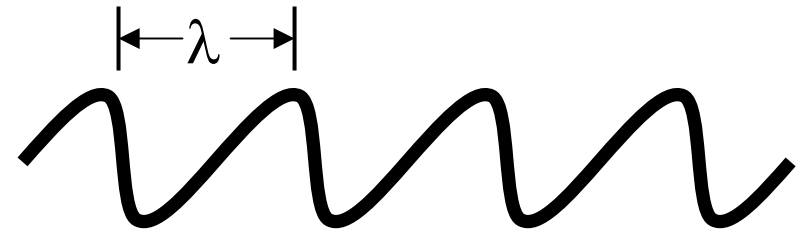
# Nonlinear Harmonic Generation

- FEL instability creates energy and density modulation at  $\lambda$ ,
- Near saturation, strong bunching at fundamental  $\lambda$  produces rich harmonic components

small signal, linear regime



near saturation, nonlinear regime



- Coherent harmonics drive by fundamental  $\lambda$ 
  - gain length =  $L_G/h$  ( $h$  is harmonic order)
  - transverse coherence
  - temporal structures

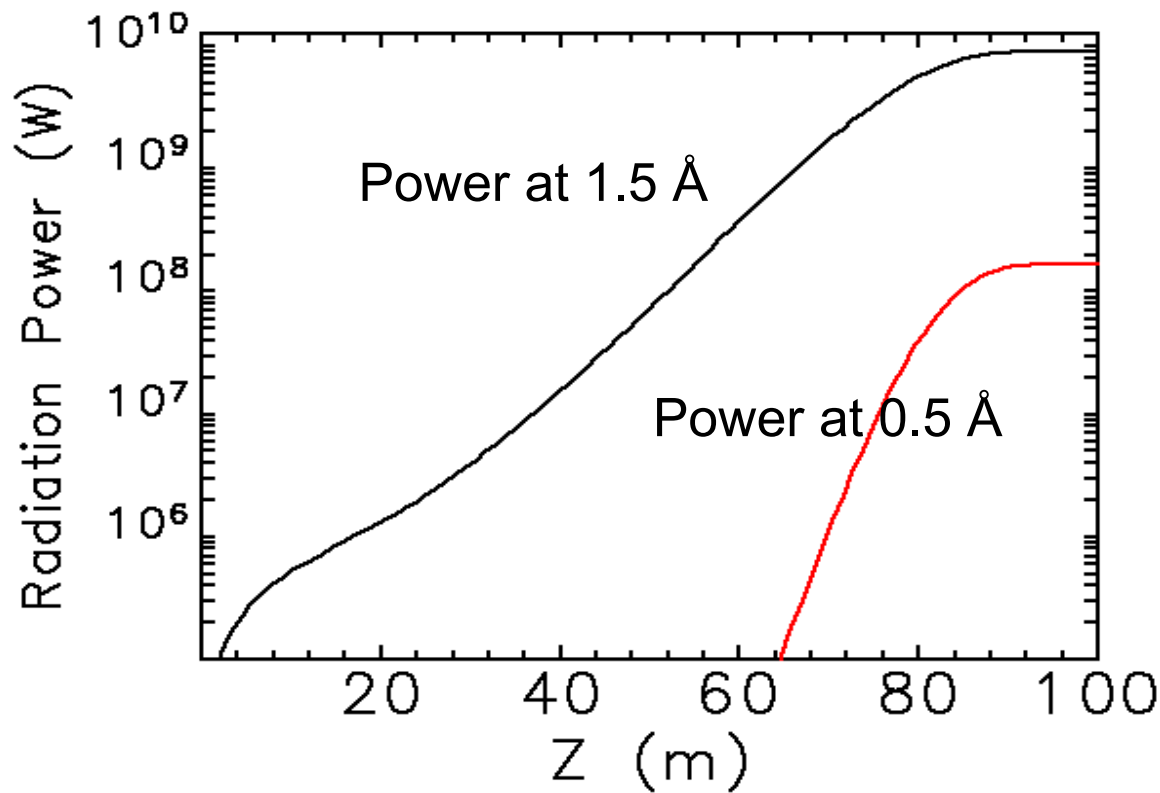




## Plenty of Power at (3X) Shorter Wavelength

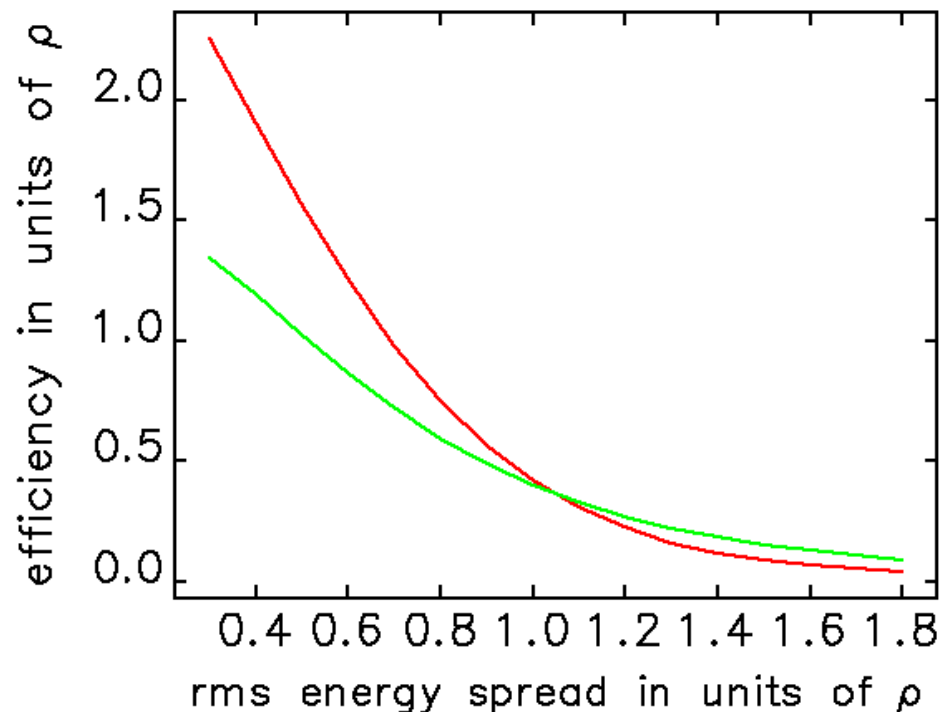
- Theory predicts third harmonic reaches 1% of fundamental, Verified by recent high-gain experiments

LCLS expectation



## Saturation Mechanism

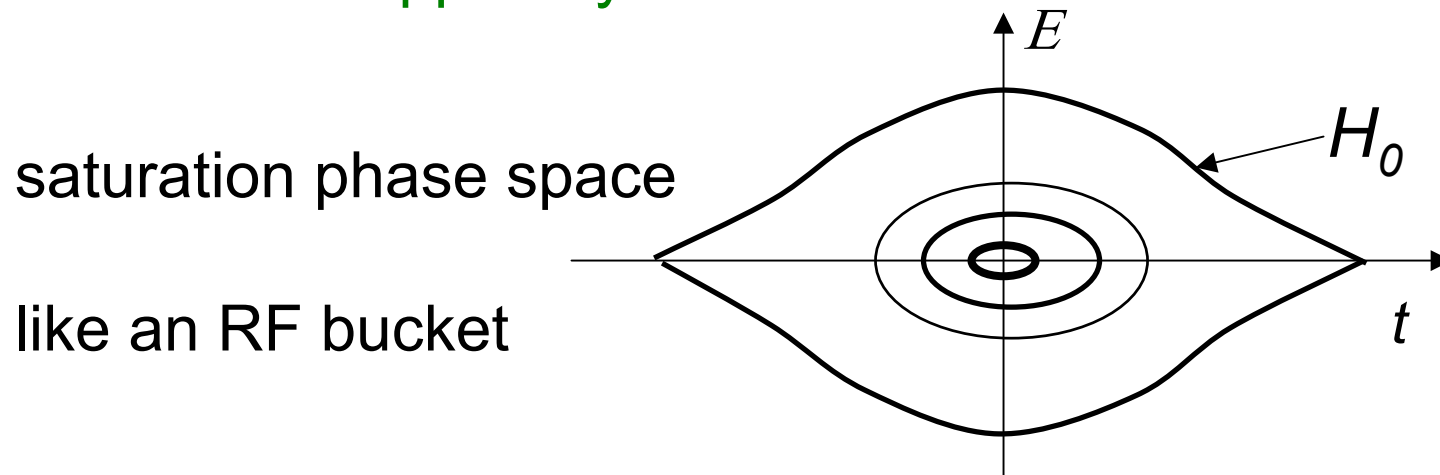
- Quasi-linear relaxation: strong radiation field modifies e-beam distribution → increased energy spread suppress the gain → FEL saturation



- Quasi-linear solution
- Simulation fitting (Xie)

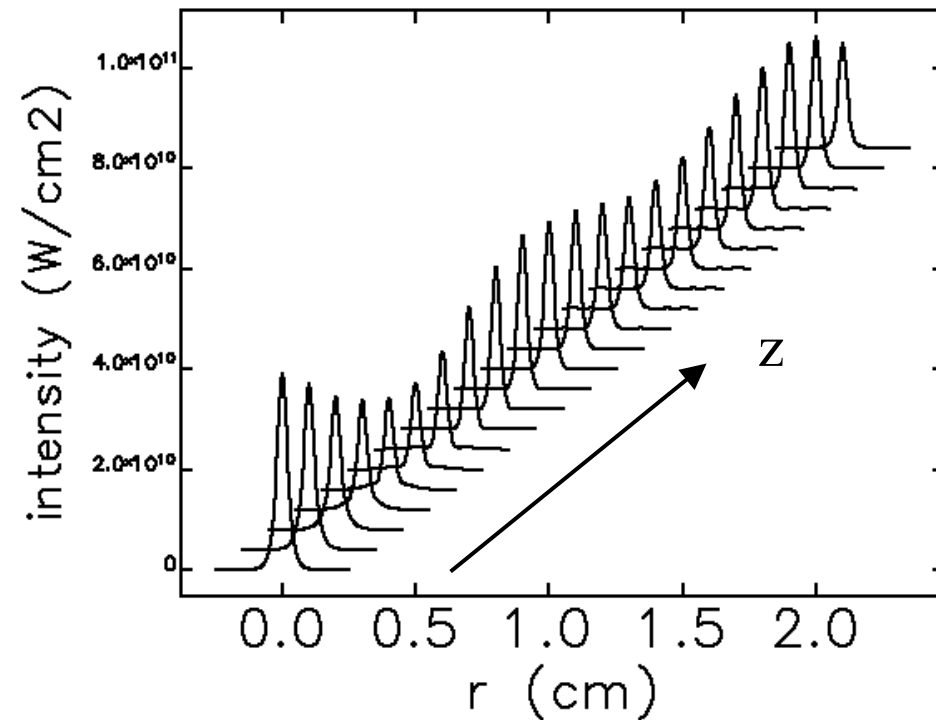
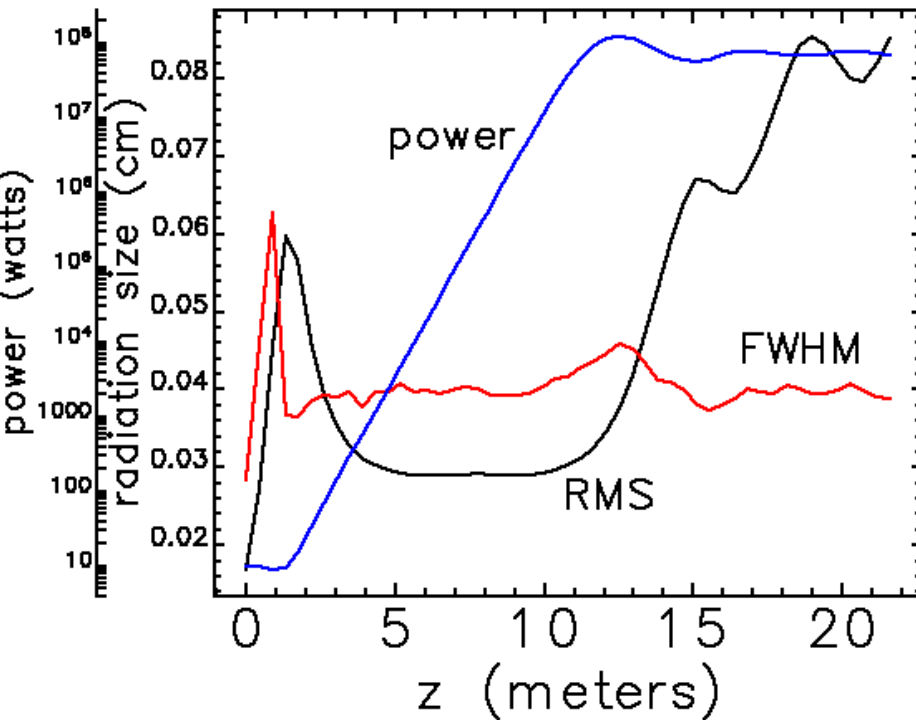
## Saturation Behaviors

- XFELs operate in saturation for max. power/stability, seeding schemes go deep saturation to reduce fluctuation
- Electrons trapped by combined radiation+undulator fields



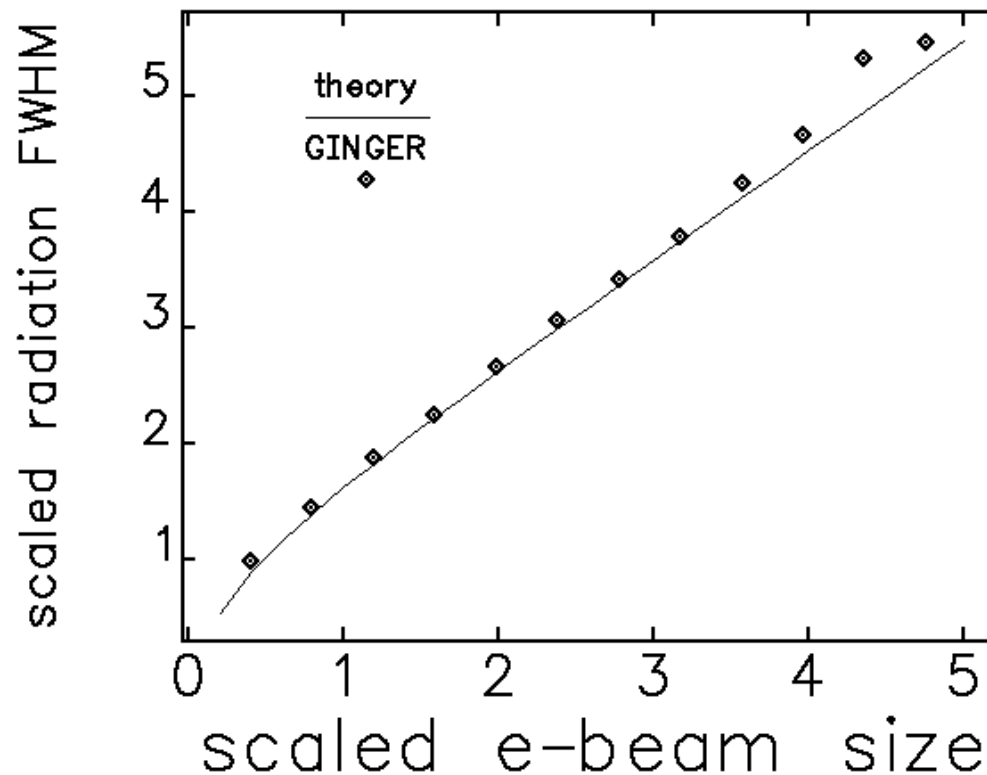
- Radiation power stays roughly constant, but phase advances due to the beam-radiation interaction  
→ an effective index of refraction ( $>1$ ) (Scharleman et al.)

# Refractive Guiding



- Guided mode that carries fixed power → constant FWHM  
some excess power diffracts out → increased rms size  
other excess power stays oscillatory

## Guided Mode after Saturation

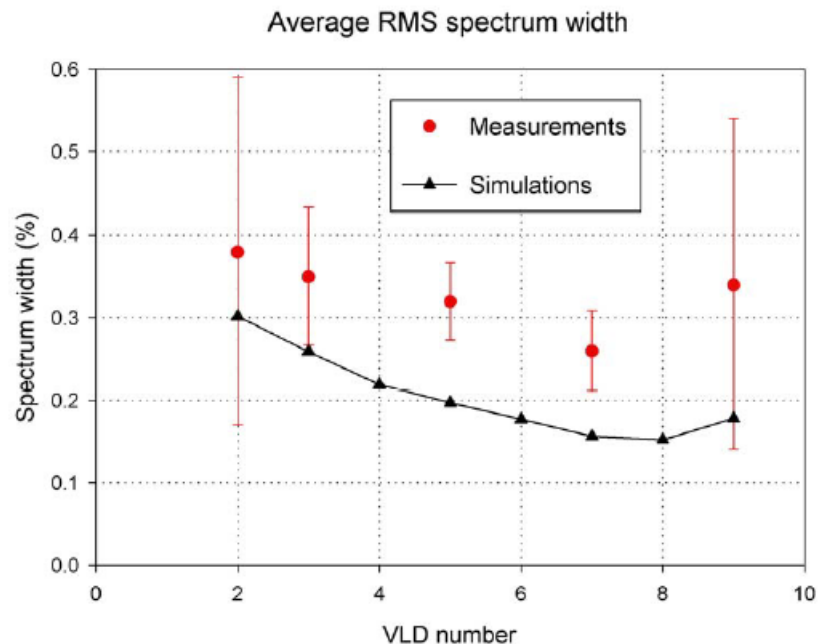


- Valid when emittance  $< \lambda/4\pi$
- For XFELs, emittance  $> \lambda/4\pi$ , any guiding?

# Sideband Instability

- Before saturation, SASE spectrum undergoes gain narrowing
- After saturation, spectrum redshifts and broaden because electron's synchrotron motion in the bucket generates sidebands

• LEUTL shows such a behavior (Sajaev et al.)





## Conclusion

- Evolution of FEL fundamental and harmonic radiation can be completely determined for simple e-beam distributions from start-up to near saturation
- Some understandings of saturation behaviors, more needed in combination with numerical simulations
- Quantum effects (Schroeder et al.) are negligible in XFELs except for quantum fluctuation due to spontaneous radiation (Saldin et al.)
- Excitements of XFELs lead to further progress in all areas of high-gain FEL research (including theory)